VIBRATION OF SYMMETRIC COMPOSITE LAMINATES IN FLUIDS

by
RAMDASS K.

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

JULY, 1987

VIBRATION OF SYMMETRIC COMPOSITE LAMINATES IN FLUIDS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

by
RAMDASS K.

to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
JULY, 1987

CERTIFICATE

This is to certify that the thesis entitled 'VIBRATION OF SYMMERTRIC COMPOSITE LAMINATES IN FLUIDS' by RAMDASS K., is a bonafide record of work done by him under our guidance and supervision, for the award of the degree of Master of Technology at the Indian Institute of Technology, Kanpur. The work carried out in this thesis has not been submitted elsewhere for the award of a degree.

(Dr. V.SUNDARARAJAN)
Professor,
Dept. of Mech. Engg.,
I.I.T. Kanpur.

Bhupinder Pal Singh.
(Dr. BHUPINDER PAL SINGH
Assistant Professor,
Dept. of Mech. Engg.,
I.I.T. Kanpur.

(THESIS SUPERVISORS)

ACKNOWLEDGEMENTS

I acknowledge with sincerity and gratitude, the guidance provided by Dr. V. Sundararajan and Dr. Bhupinder Pal Singh in sorting out various problems through the course of this thesis. I also thank them for their contributions made by way of timely advices and criticisms.

July 10, 1987
I.I.T. KANPUR

RAMDASS K.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

a,b,c	:	Half dimensions of plate and fluid elements in x,y and z directions
a,b	:	Length and bredth of the plate in the x and y-directions respectively
$\{b\}$	:	Boundary force matrix of the plate
D	:	Flexural Regidity of the plate
$[D]$	:	Bending stiffness matrix
E	:	Young's modulus of elasticity
$\{f\}$	:	Load matrix of the plate
G	:	Shear Modulus of elasticity
h	:	Thickness of the plate
$\{g\}$	:	Boundary matrix of the fluid
$[H]$	:	Fluid-stiffness matrix
$[K]$	:	Plate stiffness matrix
$[L]$	:	Interaction matrix
L	:	Lagrangian
$[M]$	:	Plate mass matrix
M_x, M_y, M_{xy}	:	Bending, Twisting moments per unit length
$[M_A]$	:	Added mass matrix
$\lfloor N \rfloor$	:	Vector of plate shape functions
p	:	Fluid pressure
$\lfloor P \rfloor$	:	Vector of fluid shape functions
$[Q]$	:	Stiffness matrix for specially orthotropic Lamina

$[\bar{Q}]$	:	Stiffness matrix for generally orthotropic lamina
Q_x, Q_y	:	Shear forces/unit length along y and x-edges of the plate respectively
q	:	Load on the plate
T	:	Kinetic Energy
U	:	Strain Energy
V	:	Work done by the external forces
w	:	Lateral diflection of the plate
μ	:	Mass/unit area of the plate
ρ	:	Density of plate material
ρ_f	:	Density of fluid
ν	:	Poisson's ratio
θ	:	Fiber orientation
ω	:	Circular frequency
λ_j	:	Non-dimensionalised frequency parameter

SUBSCRIPTS AND SUPERSCRIPTS :

(b)	:	Boundary parameters of the nodes
(be)	:	Boundary parameters of the nodes of the element
(e)	:	Element
L	:	Longitudinal
(n)	:	Nodal parameters
(ne)	:	Nodal parameters of the Element
T	:	Transverse

x,y,z	:	x,y and z derivitives
.	:	Velocity (first time derivitive)
..	:	Acceleration (Second time derivitive)
[ ]	:	Square or rectangular matrix
{ }	:	Column matrix
⌊ ⌋	:	Row matrix

ABSTRACT

A study of the vibration of thin laminates in fluid is done by the finite element method. The analysis is concerned with symmetric orthotropic laminates. FEM is applied to both the plate and the fluid domains. The fluid is considered to be ideal thus offering no damping on the vibrations.

The effect of laminate orientations, boundary conditions and the aspect ratio on the frequencies of the laminates in fluid is studied. Effect of change in fluid density on the frequencies is investigated. The in vacuo results of the laminates are also presented.

A study of the 12 d.o.f. and 16 d.o.f. elements and the relevant B.Cs. is made and 16 d.o.f. element with implied B.Cs. is found to be better.

A better 13-noded fluid element for gradual coarsening of fluid mesh is generated by static condensation instead of symmetric transformation hitherto used by many investigators. This is employed in all the investigations.

CHAPTER -1

INTRODUCTION

Natural frequencies of the underwater structures such as submerged pipelines, dams, ship hulls etc. are significantly affected by the presence of the fluid around them. The inertia of the surrounding fluid reduces the natural frequencies of the structures. The density of the surrounding fluid is the chief parameter affecting the frequencies. In addition, the geometry of the structure has an important bearing on the vibrational characteristics. Vibration o plates has an important application in the ship hull vibration. With the advent of composites, these materials are increasing used in the marine structures, especially ships and boats. Very few studies have been carried out on these structures. The present study concentrates on the symmetric orthotropic liminates in fluid. The study is limited to thin plates.

The surrounding fluid is assumed be to incompressible and non viscuous. Hence the fluid doesnot contribute to the damping of vibrations.

1.1 REVIEW OF PREVIOUS WORK :

A good review of the literature on the structural vibration in fluid is given by Muthuveerappan [1].

One of the earliest study on the vibration of plates in water was by Lamb [2] which was followed by the work of Lewis [3] who extended the analysis to the ship hull vibrations. Lamb studied a clamped circular plate in contact with infinite expanse of water on one side by Rayleigh method and suggested a formula for fundamental frequency parameter. He extended the study to fluid on two sides also. The results were proved by Powell [4] through experimental studies.

Babayev [5] studied the vibration of a cantilever plate in water. Lindholm et al [6] compared the experimental results of the vibration of cantilever plates in fluid with the theoretical predictions based on the thin plate theory. Clough [7], Malhotra [8], Asghar [9] and Fritz [10] did the analysis through empirical formulae and the conclusions of their work were used by [1] for comparison.

One of the earliest finite element formulations was by Zienkiewicz et al [11] who analysed the vibration of complex structures in fluid. Zienkiewicz and Newton [12] extended their studies to the coupled vibration with the inclusion of the compressibility of the fluid. Holand [13] extended the analysis to some two dimensional interaction problems by developing a few different types of fluid elements. Nath [14,15] performed a finite element analysis of vibration of high gravity dams due to the earth quake motion and

calculated the hydrodynamic pressure on the dams due to this motion. Most of the investigaters mentioned above.idealised the fluid as two dimensional region.

One of the earliest study including the three dimensional analysis of the fluid domain was performed by Chowdhury [16,17]. He employed the finite element method for both the plate and fluid domains. He solved for the vibration of a steel cantilever plate in water which was assumed to be non viscous and incompressible. He compared his results with that obtained by strip theory [6] and from experimental studies. A similar study with compressibility of fluid included has been performed by Marcus [18], Aggarwal et al [19] extended the analysis of Chowdhury [16] to the simply supported plate in water.

A detailed investigation on the pattern outlined by Chowdhury was carried out by Muthuveerappan et al [20,21]. They have presented the variation of frequency parameter with fluid density for various plates [21]. They later on extended the analysis to Skew plates in water [22,23].

1.2 OBJECTIVE AND SCOPE OF PRESENT WORK :

The aim of the present work is to analyse the vibration of symmetric composite laminates in fluid by the finite element method. The surrounding medium is assumed to be ideal (Nonviscous and incompressible). Thin and small deformation plate theory is employed for the analysis.

In the second chapter finite element formulation for both the plate and fluid region is given. The coupling of the plate and fluid is discussed. The generation of the added mass matrix is outlined.

In the third chapter results for the isotropic as well as the symmetric composite laminates vibrating in fluid are given. Comparison of the 12 d.o.f. and 16 d.o.f. elements and the relavent boundary conditions is done first. The effect of the laminate orientations, fluid density and the aspect ratio is discussed.

Conclusions are given in last chapter.

CHAPTER -2

FINITE ELEMENT FORMULATION

This chapter outlines the procedure employed in solving the vibration of isotropic and composite plates in fluid. Equations of motion of plate are obtained by Hamilton's principle and fluid equilibrium equation by Rayleigh Ritz method. The procedure is known in literature [16,21], but has been included here for the sake of completeness.

2.1 FEM EQUATIONS FOR PLATE :

A rectangular plate finite element with the forces and moments acting on it is shown in the Fig. 1. Q_x, Q_y are the shear forces (per unit length) along the y and x-edges of the plate respectively, M_x, M_y are the bending moments (per unit length) acting on the edges parallel to the y- and x-axes respectively ; and M_{xy} is the twisting moment (per unit length).

The Lagrangian L for the element is

$$L = T - U + V \tag{2.1}$$

where

$$T = \text{Kinetic energy of the element} = \iint_A \frac{1}{2} \mu \dot{w}^2 \, dx \, dy \tag{2.2}$$

$$U = \text{Strain energy of the element} = \iint_A \frac{1}{2} \lfloor M \rfloor \{K\} \, dx \, dy \tag{2.3}$$

V = Work done by applied forces on the element

$$= \iint_A qw \, dx \, dy + \oint_B [w(Q_x n_x + Q_y n_y) + w_x (M_x n_x + M_{xy} n_y) +$$

$$w_y (M_{xy} n_x + M_y n_y)] dB \qquad (2.4)$$

in which

w = Lateral deflection of the plate

μ = Mass/unit area of the plate

$$\lfloor M \rfloor = \lfloor M_x \; M_y \; M_{xy} \rfloor \;, \; \lfloor K \rfloor = \lfloor -w_{xx} \; -w_{yy} \; -2w_{xy} \rfloor \qquad (2.5)$$

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \,,$$

$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} \qquad (2.6)$$

$$M_x = -\left(D_{11} \frac{\partial^2 w}{\partial x^2} + D_{12} \frac{\partial^2 w}{\partial y^2} + 2 D_{16} \frac{\partial^2 w}{\partial x \, \partial y}\right).$$

$$M_y = -\left(D_{12} \frac{\partial^2 w}{\partial x^2} + D_{22} \frac{\partial^2 w}{\partial y^2} + 2D_{26} \frac{\partial^2 w}{\partial x \, \partial y}\right), \qquad (2.7)$$

$$M_{xy} = -\left(D_{16} \frac{\partial^2 w}{\partial x^2} + D_{26} \frac{\partial^2 w}{\partial y^2} + 2D_{66} \frac{\partial^2 w}{\partial x \, \partial y}\right),$$

where D_{11}, D_{12}, D_{66} are the co-efficients of the bending stiffness matrix $[D]$ of symmetric orthotropic laminate [24].

$$[D] = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \qquad (2.8)$$

The procedure for obtaining these coefficients is given in Section 2.3. For isotropic plate the above matrix becomes.

$$[D] = \frac{Eh^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \qquad (2.9)$$

ν =Poisson's ratio

h = Thickness of the plate

E = Young's modulus of the plate material

Hence,

$$L= \iint_A \frac{1}{2} \mu \ \dot{w}^2 \ dx\, dy - \iint_A \frac{1}{2} \lfloor M \rfloor \{K\} \ dx\, dy + \iint_A qw \ dx\, dy + \oint_B [Q_x n_x + Q_y n_y) + w_x (M_x n_x + M_{xy} n_y) + w_y (M_{xy} n_x + M_y n_y)] dB \qquad (2.10)$$

The highest order of derivitive in the double integrals in above Eq..(2.10) is two. Hence compatibility upto the first order derivitive is needed.

The highest order of derivitive in the same Eq.(2.10) is three. Hence completeness upto the third order derivitive is required.

The polynomial satisfying the above requirements of compatibility and completeness for rectangular element (Fig.2) is bi-cubic polynomial whose coefficients are evaluated with

four degrees of freedom (w, w_x, w_y, w_{xy}) at each node, and is well known [25, 26].

$$w^{(e)} = \lfloor N \rfloor \{w\}^{(ne)} \tag{2.11}$$

where $\lfloor N \rfloor = \lfloor N_1 \; N_2 \ldots\ldots\ldots N_{16} \rfloor$

$$\lfloor w \rfloor^{(ne)} = \lfloor w_1, w_{x1} w_{y1}, \; w_{y1}, w_{xy1}, w_2, \ldots\ldots w_{xy4} \rfloor \tag{2.12}$$

Introducing Eq.(2.11) and

$$\{M\} = [D] \{K\}$$
$$\{K\} = [B] \{w\}^{(ne)} \tag{2.13}$$

where

$$[B] = - \begin{bmatrix} N_{1,xx} & N_{2,xx} & \ldots\ldots\ldots N_{16,xx} \\ N_{1,yy} & N_{2,yy} & \ldots\ldots\ldots N_{16,yy} \\ 2N_{1,xy} & 2N_{2,xy} & \ldots\ldots\ldots 2N_{16,xy} \end{bmatrix} \tag{2.14}$$

in Eq.(2.10), Lagrangian becomes.

$$L = \frac{1}{2} \lfloor \dot{w} \rfloor^{(ne)} [M]^{(e)} \{\dot{w}\}^{(ne)} - \frac{1}{2} \lfloor w \rfloor^{(ne)} [K]^{(e)} \{w\}^{(ne)} + \lfloor w \rfloor^{(ne)} \{f\}^{(ne)} + \lfloor w \rfloor^{(ne)} \{b\}^{(be)} \tag{2.15}$$

where $[M]^{(e)}$ = mass matrix of the element = $\iint_A \mu \{N\} \lfloor N \rfloor \, dx \, dy$

$[K]^{(e)}$ = stiffness matrix of the element =

$$\iint_A [B]^T [D] [B] \, dx \, dy \tag{2.16}$$

$\{f\}^{(ne)}$ = load matrix of the element = $\iint_A q \{N\} \, dx \, dy$

$\{b\}^{(be)}$ = boundary force matrix of the element =

$$\oint_B ((Q_x n_x + Q_y n_y) \{N\} + (M_x n_x + M_{xy} n_y) \{N,_x\} + (M_{xy} n_x + M_y n_y) \{N,_y\}) dB \qquad (2.17)$$

Applying Hamiton's principle, equations of motion are obtained :

$$[M]^{(e)} \{\ddot{w}\}^{(ne)} + [K]^{(e)} \{w\}^{(ne)} = \{f\}^{(ne)} + \{b\}^{(be)} \qquad (2.18)$$

The assembled equations for the plate are

$$[M] \{\ddot{w}\}^{(n)} + [K] \{w\}^{(n)} = \{f\}^{(n)} + \{b\}^{(b)} \qquad (2.19)$$

where $[K]$, $[M]$, $\{f\}^{(n)}$, $\{b\}^{(b)}$ are the assembled matrices.

2.2 NOTE ON 12 d.o.f. ELEMENT :

In addition to the 16 d.o.f. element discussed in Section 2.1, the 12 d.o.f. element with w, w_x and w_y as nodal degrees of freedom is popularly used in static and dynamic analysis of plate. This element does not satisfy the compatibility of w_y along the x-edge and w_x along the y-edge. For static analysis a comparitive study of these elements is well documented, see [25,26]. It is shown that 16 d.o.f. element

gives much better results than 12 d.o.f. element. Comparisons between these two elements have been done for equal number of elements and not for the comparable size of the solution matrix, thus giving the impression that for practically acceptable accuracies 12 d.o.f. element is computationally cheaper.

It seems no comparitive study has been done to study these two elements for dynamic analysis of plates. The present work includes this study.

2.3 BENDING STIFFNESS MATRIX FOR ORTHOTROPIC LAMINATE :

For specially orthotropic Lamina, the stiffness matrix is

$$[Q] \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{2.20}$$

when $Q_{11} = \dfrac{E_L}{(1-\nu_{LT}\,\nu_{TL})}$

$$Q_{12} = \frac{\nu_{LT}\,E_T}{(1-\nu_{LT}\,\nu_{TL})} = \frac{\nu_{TL}\,E_L}{(1-\nu_{LT}\,\nu_{TL})} \tag{2.21}$$

$$Q_{22} = \frac{E_T}{(1-\nu_{LT}\,\nu_{TL})}$$

$$Q_{66} = G_{LT}$$

E_L, E_T G_{LT} are the longitudinal, tranverse and shear modules of this orthotropic lamina and ν_{LT} and ν_{TL} are the major and minor Poisson's ratios.

Stiffness matrix for the generally orthotropic

lamina with fibre orientation at angle θ to x-axis is

$$[\bar{Q}] = [T]^{-1} [Q] [T], \tag{2.22}$$

where

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{2.23}$$

$[Q]$ is changed suitably to connect tensor strains.

For composite plate of n orthotropic laminates, bending stiffness coefficients are,

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \bar{Q}_{ij}^{(k)} (h_k^3 - h_{k-1}^3), \quad i,j = 1,2,6 \tag{2.24}$$

where h_k is the distance of the upper surface of the k^{th} laminate from the geometric mid surface.

2.4 FEM EQUATIONS FOR FLUID :

The differential equation governing the pressure distribution during small amplitude motion of an incompressible and nonviscous fluid is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0 \tag{2.25}$$

where p represents the excess pressure over the hydrostatic pressure due to the oscillation of the fluid.

The corresponding integral to be minimised is [27]

$$I = \iiint_V \frac{1}{2} \left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2 + \left(\frac{\partial p}{\partial z}\right)^2 \right] dx\, dy\, dz - \oiint_B p \frac{\partial p}{\partial n}\, dB \tag{2.26}$$

The highest order derivitive in the triple integral of Eq.(2.26) is one. Hence compatibility of p alone is required.

The highest order derivitive in the Eq.(2.26) is also one. Hence completeness upto the first derivitive of p is required.

The polynomial satisfying the above requirements of compatibility and completeness for eight noded brick element (Fig.3) is tri-linear polynomial whose coefficients are evaluated with only pressure as the nodal degree of freedom and is well known in literature [16]

$$p^{(e)} = \lfloor P \rfloor \{p\}^{(ne)} \qquad (2.27)$$

$$\lfloor P \rfloor = \lfloor P_1\ P_2 \ \ldots\ldots\ldots\ldots\ P_8 \rfloor$$

$$\lfloor p \rfloor^{(ne)} = \lfloor p_1\ p_2 \ldots\ldots\ldots\ldots\ p_8 \rfloor$$

Substituting this in the Eq.(2.26), one gets

$$I = \frac{1}{2} \lfloor p \rfloor^{(ne)} [H]^{(e)} \{p\}^{(ne)} - \lfloor p \rfloor^{(ne)} \{g\}^{(be)} \qquad (2.28)$$

where $[H]^{(e)}$ = Fluid stiffness matrix of the element =

$$\iiint_V \left(\{P,_x\} \lfloor P,_x \rfloor + \{P,_y\} \lfloor P,_y \rfloor + \{P,_z\} \lfloor P,_x \rfloor \right) dx\, dy\, dz \qquad (2.29)$$

$\{g\}^{(be)} =$ Boundary matrix of the fluid element =

$$\oiint_B \frac{\partial p}{\partial n} \{P\} \, dB \qquad (2.30)$$

Using Rayleigh Ritz method, Eq.(2.28) gives the fluid equilibrium equations

$$[H]^{(e)} \{p\}^{(ne)} = \{g\}^{(be)} \qquad (2.31)$$

Assembling the element matricies

$$[H] \{p\}^{(n)} = \{g\}^{(b)} \qquad (2.32)$$

where $[H]$, $\{g\}^{(b)}$ are assembled matrices.

In addition to the 8-noded fluid element mentioned earlier, 13-noded fluid element with pressure as the d.o.f. is employed for gradual coarsening of the fluid finite element mesh. The procedure for deriving the stiffness matrix for this element is discussed in Section (2.5).

2.5 13-NODED FLUID STIFFNESS MATRIX :

A 13-noded fluid element is shown in Fig. 4. This element has nine nodes on one surface and four on every other surface. To derive the stiffness matrix for this element, eight of the 8-noded elements are first assembled (Fig. 5) to form a (27 x 27) matrix $[H]$. Chowdhury [16] employed a transformation matrix which preserves the linear variation of pressure in the z-direction for eleminating the 14 of the

unwanted degrees of freedom. For more accurate results static condensation technique has been adopted in this work to get the required stiffness matrix from the assembled (27 x 27) $[H]$ matrix. The procedure is outlined below.

The assembled equilibrium equations for eight, 8-noded fluid elements are

$$\begin{bmatrix} \; H \; \end{bmatrix} \left\{ p \right\}^{(ne)} = \left\{ g \right\}^{(be)} \qquad (2.33)$$

Partitioning the above matrix

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}^{(ne)} = \begin{Bmatrix} g_1 \\ g_2 \end{Bmatrix}^{(be)} \qquad (2.34)$$

where $\{p_1\}^{(ne)}$ = required 13 d.o.f.

$\{p_2\}$ = Unwanted 14 d.o.f.

Hence $[H_{11}] \{p_1\}^{(ne)} + [H_{12}] \{p_2\}^{(ne)} = \{g_1\}^{(be)}$

$$(2.35)$$

$$[H_{21}] \{p_1\}^{(ne)} + [H_{22}] \{p_2\}^{(ne)} = \{g_2\}^{(be)}$$

$$(2.36)$$

From Eq. (2.36)

$$\{p_2\}^{ne)} = - [H_{22}]^{-1} [H_{21}] \{p_1\}^{(ne)} + [H_{22}]^{-1} \{g_2\}^{(be)}$$

$$(2.37)$$

Substituting in Eq. (2.35)

$$([H_{11}] - [H_{12}] [H_{22}]^{-1} [H_{21}] \{p_1\}^{(ne)} = \{g_1\}^{(be)} - [H_{12}] [H_{22}]^{-1} \{g_2\}^{(be)} \qquad (2.38)$$

Thus stiffness matrix for 13-noded element is

$$[H]^{(e)} = [H_{11}] - [H_{12}] [H_{22}]^{-1} [H_{21}] \qquad (2.39)$$

2.6 COUPLING OF PLATE AND FLUID EQUATIONS :

The equation of motion for the plate Eq. (2.19) is

$$[M] \{\ddot{w}\}^{(n)} + [K] \{w\}^{(n)} = \{f\}^{(n)} + \{b\}^{(n)} \qquad (2.40)$$

From Eq. (2.17)

$$\{f\}^{(ne)} = \iint_A \{N\} \ q \ dx \ dy$$

The pressure p of the fluid acts as a distributed force q, thus

$$\{f\}^{(ne)} = \iint_A \{N\} \ p^{(e)} \ dx \ dy \qquad (2.41)$$

Substituting $p^{(e)}$ from Eq. (2.27), one gets

$$\{f\}^{(ne)} = \iint_A \{N\} \lfloor P \rfloor \ dx \ dy \ \{p\}^{(ne)} = [L]^{(e)} \{p\}^{(ne)} \qquad (2.42)$$

where

$$[L]^{(e)} = \text{Interaction matrix for the element} =$$

$$\iint_A \{N\} \lfloor P \rfloor \; dx\,dy \qquad (2.43)$$

Assembling the $\{f\}^{(ne)}$s, one gets

$$\{f\}^{(n)} = [L] \; \{p\}^{(n)} \qquad (2.44)$$

where $[L]$ is the overall interaction matrix, and $\{p\}^{(n)}$ is the matrix of pressures at the interacting nodes.

Substituting for $\{p\}^{(n)}$ from Eq. (2.32) in Eq.(2.44), one gets

$$\{f\}^{(n)} = [L]\,[H]^{-1} \; \{g\}^{(b)} \qquad (2.45)$$

where $[H]^{-1}$ is the fluid stiffness matrix inverted after applying the boundary conditions. It may be noted that after inversion of $[H]$ matrix, only part corresponding to the interaction nodes is used in Eq. (2.45).

Matrix $\{g\}^{(be)}$ is given by Eq. (2.30) and is

$$\{g\}^{(be)} = \oint_B \{P\} \; \frac{\partial p}{\partial n} \, dB \qquad (2.46)$$

At the solid fluid interface

$$\frac{\partial p}{\partial n} = - \rho_f \, \ddot{w} \qquad (2.47)$$

where ρ_f is the density of the fluid and $\ddot{w}$ is the plate acceleration.

Using Eq.(2.46) and Eq.(2.47), one gets

$$\{g\}^{(be)} = -\rho_f \oiint_B \{P\} \ddot{w}^{(e)} dB \tag{2.48}$$

Using Eq.(2.11), this becomes

$$\begin{aligned} \{g\}^{(be)} &= -\rho_f \oiint_B \{P\} \lfloor N \rfloor dB \; \{\ddot{w}\}^{(ne)} \\ &= -\rho_f [L]^{(e)T} \{\ddot{w}\}^{(ne)} \end{aligned} \tag{2.49}$$

Assembling $\{g\}^{(be)}$ over the plate, one gets

$$\{g\}^{(b)} = -\rho_f [L]^T \{\ddot{w}\}^{(n)} \tag{2.50}$$

Substituting this in Eq.(2.45), one gets

$$\{f\}^{(n)} = \rho_f [L] [H]^{-1} [L]^T \{\ddot{w}\}^{(n)} \tag{2.51}$$

Thus Eq.(2.40) becomes

$$([M] + [M_A])\{\ddot{w}\}^{(n)} + [K] \{w\}^{(n)} = \{b\}^{(n)} \tag{2.52}$$

where $[M_A]$ is the added mass matrix

$$[M_A] = \rho_f [L] [H]^{-1} [L]^T \tag{2.53}$$

CHAPTER -3

RESULTS AND DISCUSSIONS

Firstly,a comparison of 16.d.o.f. and 12 d.o.f. elements along with relevent boundary conditions is made. This is followed by a study of isotropic plate in fluid. A detailed investigation of the symmetic composite laminates in fluid is presented thereafter.

3.1 PLATE ELEMENTS AND BOUNDARY CONDITIONS :

Frequencies for the simply supported plate employing 16 d.o.f. and 12 d.o.f. elements are given in Table 1 and 2 for classical as well as classical with implied B.Cs. Similar results for the cantilever plate are given in Tables 3 and 4, and for clamped-free-clamped free plate in Tables 5 and 6. Table 7 shows comparison of both elements for solution matrices of comparable sizes.

Before discussing these elements further, the B.Cs. are catalogued. The classical B.Cs. for various edges one :

Clamped edge : For edge x=a to be clamped

$$w\Big|_{x=a} = 0 \; ; \quad \frac{\partial w}{\partial x}\Big|_{x=a} = 0 \tag{3.1}$$

Simply supported edge : For edge x=a to be simply supported

$$w\Big|_{x=a} = 0 \, , \quad M_x\Big|_{x=a} = 0 \tag{3.2}$$

Free edge : For edge x=a to be free

$$M_x \Big|_{x=a} = 0, \quad \left(Q_x + \frac{\partial M_{xy}}{\partial y}\right) \Big|_{x=a} = 0 \tag{3.3}$$

The implied B.Cs are

Clamped edge : For x=a to be clamped

$$\frac{\partial w}{\partial y} \Big|_{x=a} = 0, \quad \frac{\partial^2 w}{\partial x \, \partial y} \Big|_{x=a} = 0 \tag{3.4}$$

Simply Supported edge : For edge x=a to be simply supported

$$\frac{\partial w}{\partial y} \Big|_{x=a} = 0 \tag{3.5}$$

The frequencies for isotropic plate are presented in the form of non-dimensionalised frequency parameter λ,

$$\lambda = \omega a^2 \sqrt{\frac{\mu}{D}} \tag{3.6}$$

Where ω = circular frequency

a = length of the plate
μ = mass per unit area of the plate = ρh
ρ = density of plate

D = flexural rigidity of plate = $\dfrac{Eh^3}{12(1-\nu^2)}$

h = thickness of plate

ν = Poisson's ratio.

Tables 2,4 and 6 show the results for the 12 d.o.f. element for different B.Cs. It is observed that the frequencies

by classical B.Cs. and classical with implied B.Cs. are essentially same. But in both these B.Cs., convergence is monotonically increasing in some modes, is monotonically decreasing in other modes; and sometimes the convergence is non-monotonic. For the simply supported case for which exact results are known it is seen that the FEM frequencies are less than the exact results.

Tables 1,3,5 show the results for the 16 d.o.f. element for classical and classical with implied B.Cs. Comparison of the results show that the frequencies are very accurate by classical with implied B.Cs. Sometimes the results with classical B.Cs. only show an appreciable error. The results for the former B.Cs. are generally found to be more accurate even with a fewer elements. The convergence is always monotonically decreasing, where as with classical B.Cs. only, convergence is sometimes monotonically increasing and some times monotonically decreasing. It may be noted that the trend of monotonic decrease in convergence is consistant with consistant FEM formulation.

From the observations above it is clear that classical with implied B.Cs gives greater accuracy for the 16 d.o.f. element. In addition introduction of implied B.Cs. reduce the size of the solution matrices thus decreasing the computational effort. Hence implied B.Cs. are always used in the subsequent discussion and results in this work.

Tables 1 and 2 show the results for the two elements for the simply supported plate for which exact solution is available. It is evident that the 16 d.o.f. element results are better than that of 12 d.o.f. elements. Even with a fewer elements, the 16 d.o.f. element gives quite accurate results. The discrepency between the two elements is appreciable in the fourth mode where the 12 d.o.f. result shows an error of 9 percent where as for 16 d.o.f. it is almost zero. With the 16 d.o.f. element the convergence is always monotonically decreasing where as for the 12 d.o.f., element convergence is monotonically increasing in some modes, monotonically decreasing in some other modes and in some modes not monotonic at all.

Tables 3 and 4 give similar results for cantilever plates. Same trends as those for simply supported plates are observed. But the results by 12 d.o.f. element are also quite accurate, thus showing that the clamped edges are less affected by the slope incompatibility of the 12 d.o.f. element than the simply supported edges.

Tables 5 and 6 show similar studies for the clamped-free-clamped-free plate, and same trends as those for the cantilever plate are observed.

Table 7 shows the comparison of the two elements for the comparable size of solution matrices. It is obvious that the 16 d.o.f. element is superior to 12 d.o.f. element computationally.

From the foregoing discussion it can be concluded that the 16 d.o.f. element with implied B.Cs. is more accurate, computationally more efficient and gives monotonically decreasing convergence. Hence this element is employed in the further investigation in this work.

3.2 COMPUTATIONAL DETAILS :

The spread of fluid around the plate and the depth below it are in reality infinite in extension. But the effect of fluid is negligible after a certain distance from the plate. Hence it is sufficient to take a finite extension of the fluid for analysis. Muthuveerappan [1] has shown that it is sufficient to take the fluid spread around the plate to be equal to the maximum plate dimension and the water depth below to be 2.75 times the maximum plate dimension.

The fluid domain with the finite element discritisation is shown in the Fig. 6. The layer by layer description of these fluid elements is shown in Fig. 7. In general 8-noded tri-linear elements have been used. For gradual coarsening of the mesh the 13-noded element derived by static condensation has been employed. Eighty eight fluid elements with 186 nodes including 51 surface nodes have been used. Out of these 88 elements, 16 elements are 13-noded.

The overall interaction matrix coupling the plate with the fluid degrees of freedom is obtain by assembling the

elementary interaction matrices of Eq. (2.43). In the presence of fluid on both sides, this matrix is evaluated considering the interfaces above and below the plate. The added mass matrix accounting for the inertia of the fluid is evaluated from the interaction matrix and the fluid stiffness matrix from Eq.(2.53). This is then added to the plate mass matrix and the resulting Eq.(2.52) is solved for the eigen values after applying the B.Cs. A NAG FO2BJF routine is employed for eigen value calculations.

3.3 ISOTROPIC PLATE IN FLUID :

Tables 8 and 9 give the results of the present work for isotropic cantilever and simply supported steel plates vibrating in water along with the earlier results. The plate is at a depth of 0.25 times the length of plate. The results of the present work include both types of plate elements previously discussed in Section 3.1. Results employing the 13-noded fluid element derived by static condensation are compared with the results by 13-noded element used by [16,21]. In vacuo results are also presented for comparison.

Table 8 gives the results for the cantilever plate. To check the present computer programs, first the results are computed for 12 d.o.f. element with 13-noded fluid element used by [16, 21]. A good agreement is found with these references. Also results are computed for this element with the more

accurate [16] present 13-noded fluid element. The frequencies are found to be slightly less than the earlier ones and results are found to be closer to the experimental value. Next, results are computed for 16 d.o.f. element with the latter fluid element, and results indicate the correctness of the present computer programes for this element.

The simply supported plate results are presented in Table 9. The results from the present work for both the types of 13-noded elements are compared with previous results [1] obtained by 12 d.o.f. element and 13 noded fluid element of [16,21]. It is observed that higher mode frequencies are slightly higher than those of the [1]. This difference is quite appreciable for the fourth mode. These trends are consistent with the in vacuo case of simply supported plates. Hence the present frequencies are more accurate.

From the discussion above it can be said that 16 d.o.f. element with 13-noded fluid element obtained by condensation gives better results. This is adopted in future investigations.

3.4 COMPOSITE SYMMETRIC LAMINATE RESULTS :

For composite laminates the frequencies are presented in the form of non-dimensionalised frequency parameter, λ,

$$\lambda = \omega a^2 \sqrt{\frac{\mu}{E_T h^3}} \qquad (3.7)$$

where E_T is the transverse modulas of the lamina.

3.4.1 Composite Plate In Vacuo Frequencies :

Table 10 lists the in vacuo composite results for

a single 0°-lamina and a 0/90/0 laminate. They have been compared with exact results, Jones [24].

The results for both lamina and the laminate are very close to the exact value even upto the fifth mode. The results obtained are slightly higher than the exact values as expected.

It is clear from the above observations that the present programme is accurate for the analysis of symmetric composite laminates.

3.4.2 Composite Laminates in Fluid :

Tables 11-13 give the in vacuo and in fluid frequencies of symmetric composite laminates for different boundary conditions and various laminate orientations. The laminates are at a depth of 0.25 times the length. A typical graphite-epoxy composite has been considered for the analysis. The properties of this are given in Table 10.

It is observed from tables that the first mode frequency decreases by 55 % and second mode by 45 % for all the laminate orientations for the three boundary conditions studied. For higher modes the decrease depends on the laminate orientations and boundary conditions.

3.4.3 Variation in Frequencies with change in Fluid Density :

Table 14 shows the effect of fluid density on frequencies for the square contilever laminate under investigation.

As expected, the effect of the fluid decreases with decrease in fluid density. This is due to the fact that the inertia of the fluid on the plate decreases with decreasing density of the fluid thus lessening the added mass on the plate. From the table it is clear that the lower modes are affected to a greater extent than the higher modes. The first mode is the most affected with a reduction in frequency of 60 % for plate to fluid density ratio of one which gradually decreases for higher ratios and is 24 % for a ratio of 8. Similar trend is observed for higher modes.

3.4.4 Variation in Frequencies with Aspect Ratios (b/a) :

Table 15 shows the effect of the aspect ratio on the frequencies of a simply supported graphite epoxy laminate. From the table it is obvious that with decrease in the aspect ratio the effect of fluid on the frequencies increases for the first two and fourth modes. No particular trend is noticed for the other modes.

Table 1 : Simply Supported Plate - In Vacuo Frequencies - 16 d.o.f. Element

No. of Elements / λ	Classical with Implied B.C. 4	9	16	Exact [28]	Classical B.C. 16	9	4
λ_1	19.782	19.748	19.742	19.74	19.711	19.624	18.916
λ_2	52.991	49.737	49.476	49.35	49.188	48.568	40.477
λ_3	52.991	49.737	49.476	49.35	49.188	48.568	40.477
λ_4	83.905	79.486	79.127	78.96	77.356	71.786	41.985
λ_5	118.701	107.676	100.186	98.70	98.787	97.033	73.813

Table 2 : Simply Supported Plate - In Vacuo Frequencies - 12 d.o.f. Element

	Classical with Implied B.C.				Classical B.C.		
No. of Elements / λ	4	9	16	Exact [28]	16	9	4
λ_1	18.021	18.776	19.150	19.74	19.146	18.758	17.845
λ_2	50.911	46.594	47.405	49.35	47.342	46.289	42.644
λ_3	50.911	46.594	47.405	49.35	47.342	46.289	42.644
λ_4	108.561	70.144	72.085	78.96	71.713	67.896	43.092
λ_5	124.413	105.264	96.218	98.70	95.806	94.676	67.788

Table 3: Cantilever Plate - In Vacuo Frequencies - 16 d.o.f. Element

No. of Elements / λ	Classical with Implied B.C.			Approximate [28]	Classical B.C.		
	4	9	16		16	9	4
λ_1	3.486	3.478	3.475	3.492	3.355	3.260	3.026
λ_2	8.567	8.528	8.518	8.525	8.163	8.062	7.982
λ_3	21.476	21.373	21.325	21.43	20.517	20.189	19.651
λ_4	27.415	27.335	27.254	27.33	27.175	27.242	27.330
λ_5	31.530	31.156	31.040	31.11	29.749	29.459	28.683

Table 4 : Cantilever Plate - In Vacuo Frequencies - 12 d.o.f. Element

	Classical with Implied B.C.				Classical B.C.		
No. of Elements / λ	4	9	16	Approximate [28]	16	9	4
λ_1	3.457	3.467	3.469	3.492	3.463	3.456	3.434
λ_2	8.551	8.528	8.520	8.525	8.494	8.489	8.471
λ_3	21.767	21.680	21.538	21.43	21.491	21.612	21.676
λ_4	26.333	26.853	26.994	27.33	26.987	26.843	26.321
λ_5	30.231	30.802	30.914	31.11	30.824	30.658	29.533

Table 5: Clamped-Free-Clamped-Free Plate-In Vacuo Frequencies- 16 d.o.f. Element

No. of Elements / λ	Classical with Implied B.C. 4	9	16	Approximate [28]	Classical B.C. 16	9	4
λ_1	22.580	22.302	22.228	22.27	20.717	19.997	18.866
λ_2	27.052	26.586	26.492	26.53	23.974	23.330	22.265
λ_3	44.855	44.058	43.792	43.66	42.326	42.493	44.772
λ_4	81.587	62.459	61.803	61.47	58.083	56.773	47.772
λ_5	87.351	68.711	67.854	67.55	61.974	60.466	55.934

Table 6 : Clamped-Free-Clamped-Free Plate - In Vacuo Frequencies -12 d.o.f. Element

	Classical with Implied B.C.				Classical B.C.		
No. of Elements / λ	4	9	16	Approximate [28]	16	9	4
λ_1	22.212	22.438	22.376	22.27	22.302	22.333	22.048
λ_2	25.623	26.251	26.373	26.53	26.189	25.962	24.666
λ_3	43.384	41.779	42.178	43.66	42.003	41.493	43.355
λ_4	81.974	62.051	62.141	61.47	61.999	61.709	48.594
λ_5	84.669	65.207	66.777	67.55	66.388	64.210	56.210

Table 7 : Comparison of In Vacuo Frequencies by two Elements for Comparable the Solution Matrices.

	Simply Supported			Cantilever			Clamped-Free-Clamped-Free		
λ	12.d.o.f. 16 Elements	16 d.o.f. 9 Elements	Exact [28]	12 d.o.f. 16 Elements	16 d.o.f. 9 Elements	Approximate [28]	12 d.o.f. 16 Elements	16 d.o.f. 9 Elements	Apprximate [28]
Solution matrixSize	39x39	36x36		60x60	48x48		45x45	32x32	
λ_1	19.150	19.748	19.74	3.469	3.478	3.492	22.376	22.302	22.27
λ_2	47.405	49.737	49.35	8.520	8.528	8.525	26.373	26.586	26.53
λ_3	47.405	49.737	49.35	21.538	21.373	21.43	42.178	44.058	43.66
λ_4	72.085	79.486	78.96	26.994	27.335	27.33	62.141	62.459	61.47
λ_5	96.218	107.676	98.70	30.914	31.156	31.11	66.777	68.711	67.55

Table 8: Comparison of Frequencies of a Square Cantilever Isotropic Plate in water. ρ=7861.11 kg/m^3, ρ_f=1000 kg/m^3, (h/b)=0.0131. All results for 16 Elements.

λ	Muthuveerappan et al [21]	Chowdhury [16]	Experimental [16]	Present work			
				12.d.o.f.		16 d.o.f.	
				[16] 13-noded fluid element	Present 13-noded fluid element	Present 13-noded fluid element	In Vacuo
λ_1	1.597	1.542	1.475	1.575	1.481	1.484	3.475
λ_2	4.76	4.72	4.50	4.730	4.414	4.414	8.518
λ_3	11.65	11.52	10.75	11.630	11.135	11.066	21.325
λ_4	17.53	17.50	14.75	17.494	16.696	16.860	27.253
λ_5	19.36	19.30	17.00	19.141	18.195	18.339	31.040

Table 9 : Comparison of Frequencies of Square Simply Supported Isotropic Plate. Data as in Table 8.

λ	Muthuveerappan [1,22] 12 d.o.f.	Present work 16 d.o.f. [16] 13-noded fluid Element	Present work 16 d.o.f. Present 13-noded Fluid Element	In Vacuo Exact [28]
λ_1	7.86	8.039	7.856	19.74
λ_2	23.92	24.949	24.495	49.35
λ_3	24.37	25.518	24.916	49.35
λ_4	41.23	45.554	44.786	78.96
λ_5	-	60.645	59.752	98.70

Table 10 : In Vacuo Frequencies of a Square Simply Supported Composite Plate. E_L=20GPa, E_T=2GPa, G_{LT}=0.7GPa, ν_{LT}=0.35.

λ	Single 0° Lamina		0/90/0 Laminate	
	FEM	Exact [24]	FEM	Exact [24]
λ_1	10.371	10.369	10.371	10.369
λ_2	16.830	16.797	18.019	17.979
λ_3	30.446	30.032	33.975	33.482
λ_4	37.443	37.304	36.886	36.749
λ_5	41.616	41.476	41.616	41.476

Table 11: Frequencies of Square Cantilever Graphite Epoxy Laminate.

E_L=159 GPa, E_T=10.9 GPa, G_{LT}=6.4 GPa, ν_{LT}= 0.38,
a =4m, h =0.3m, ρ=1610 kg/ m^3, ρ_f=1000 kg/m^3.

Laminate Orientations λ	In Vacuo					In Fluid				
	0/0/0	30/0/30	45/0/45	60/0/60	90/0/90	0/0/0	30/0/30	45/0/45	60/0/60	90/0/90
λ_1	3.893	2.395	1.788	1.431	1.249	1.770	1.125	0.829	0.655	0.568
λ_2	5.004	5.161	4.867	4.109	3.164	2.779	2.780	2.656	2.265	1.731
λ_3	9.752	11.193	10.725	8.811	7.830	6.483	7.279	6.259	4.860	4.233
λ_4	20.381	15.436	12.397	12.252	11.508	13.150	9.013	7.731	7.724	7.163
λ_5	24.418	20.630	19.538	20.267	22.04	15.329	12.999	12.393	13.137	13.964

Table 12 : Frequencies of Square Simply Supported Graphite Epoxy Laminate Data as in Table 11

Laminate Orientations λ	In Vacuo					In Fluid				
	0/0/0	30/0/30	45/0/45	60/0/60	90/0/90	0/0/0	30/0/30	45/0/45	60/0/60	90/0/90
λ_1	12.376	12.695	12.931	12.712	12.376	5.259	5.456	5.567	5.462	5.258
λ_2	18.791	22.943	24.727	24.076	20.384	10.031	12.464	13.469	12.912	10.705
λ_3	32.196	37.906	37.039	38.680	37.135	20.786	20.831	19.701	20.745	23.464
λ_4	45.145	39.313	40.014	40.203	44.448	23.679	23.838	24.757	24.842	23.675
λ_5	49.658	53.471	59.265	54.920	49.658	29.603	33.872	39.336	34.731	29.588

Table.13 : Frequencies of Square Simply Supported-Clamped-Free-Clamped Graphite Epoxy Laminate,Data as in Table 11

Laminate Orientation / λ	In Vacuo					In Fluid				
	0/0/0	30/0/30	45/0/45	60/0/60	90/0/90	0/0/0	30/0/30	45/0/45	60/0/60	90/0/90
λ_1	7.025	9.414	12.763	17.451	24.530	3.205	4.333	5.937	8.132	10.754
λ_2	18.773	19.455	21.023	23.140	26.409	10.758	10.643	11.334	12.286	14.288
λ_3	20.174	26.205	33.880	34.601	33.440	10.841	15.081	20.744	22.002	21.409
λ_4	29.905	33.736	37.334	48.192	49.154	18.767	21.232	22.625	28.934	35.535
λ_5	36.618	47.418	51.381	54.997	67.943	24.385	31.623	32.586	36.052	37.890

Table 14 : Effect of Fluid density on Frequencies of Square Cantilever Graphite epoxy Laminate (45/0/45) in Fluid. Data as in Table 11.

ρ/ρ_f / λ	1.0	2.0	4.0	5.0	8.0	∞ (In Vacuc
λ_1	0.682	0.901	1.138	1.213	1.358	1.788
λ_2	2.224	2.858	3.481	3.663	4.001	4.867
λ_3	5.285	6.706	8.042	8.419	9.099	10.720
λ_4	6.628	8.221	9.639	10.028	10.722	12.397
λ_5	10.643	13.167	15.387	15.990	17.656	19.538

Table 15 : Frequencies of Rectangular Simply Supported Graphite Epoxy Laminate (45/0/45). E_L=159GPa, E_T=10.9 GPa, G_{LT}=6.4 GPa, ν_{LT}=0.38, ρ=2000 kg/m^3, ρ_f=1000 kg/m^3 and (h/b)=0.075.

(b/a) / λ	In Vacuo			In Fluid		
	1.0	0.75	0.5	1.0	0.75	0.
λ_1	12.931	17.668	30.913	6.070	7.698	12.
λ_2	24.927	32.277	47.403	14.503	17.634	23.
λ_3	37.039	52.322	72.011	21.244	29.742	42.
λ_4	40.014	52.726	102.308	26.382	33.309	59.6
λ_5	59.455	76.624	111.640	41.695	49.630	76.8

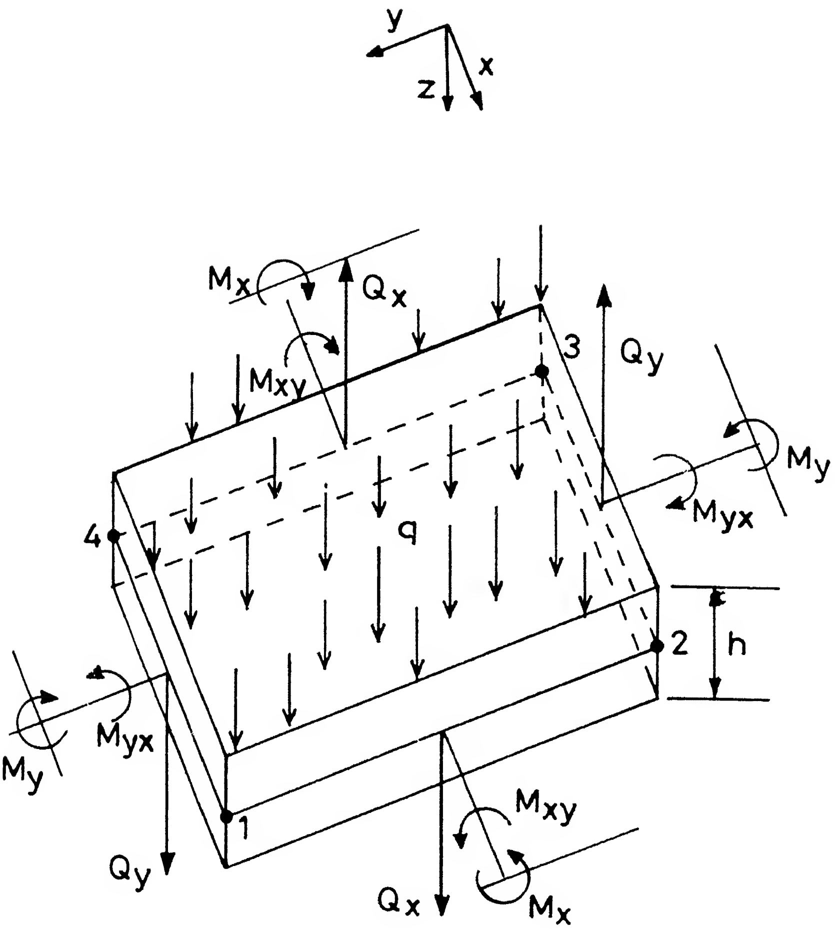


Fig. 1 Forces and moments on a rectangular plate element.

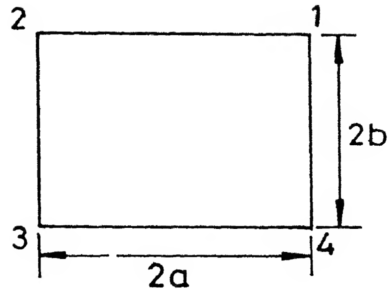


Fig. 2 Rectangular plate element.

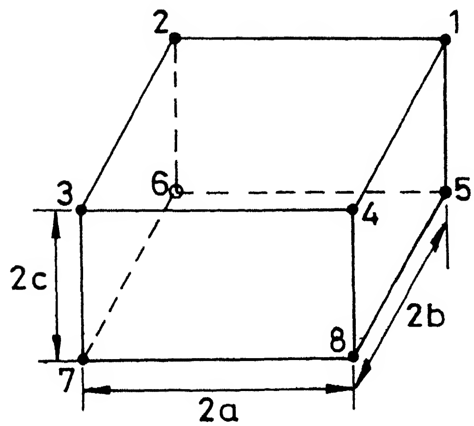


Fig.3 8-Noded fluid element.

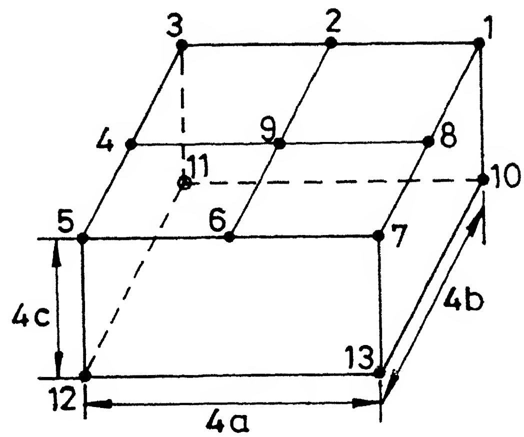


Fig. 4 13-Noded fluid elemen

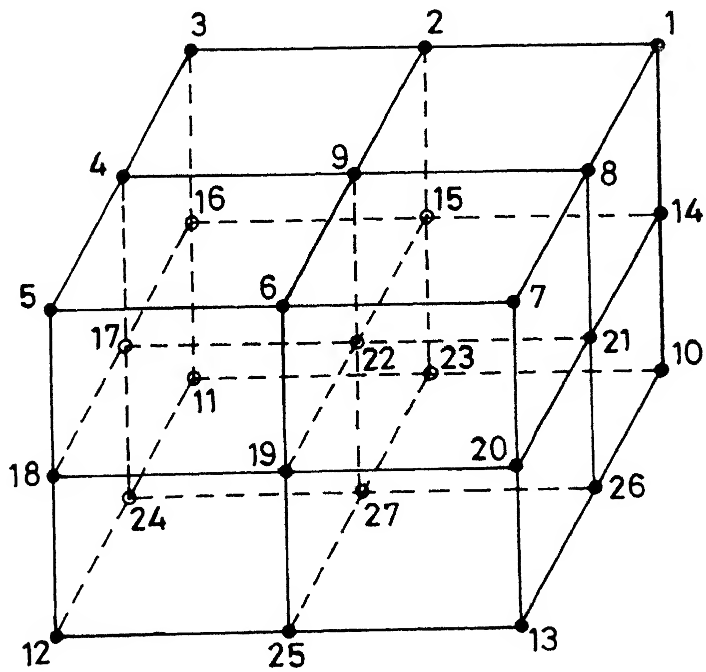
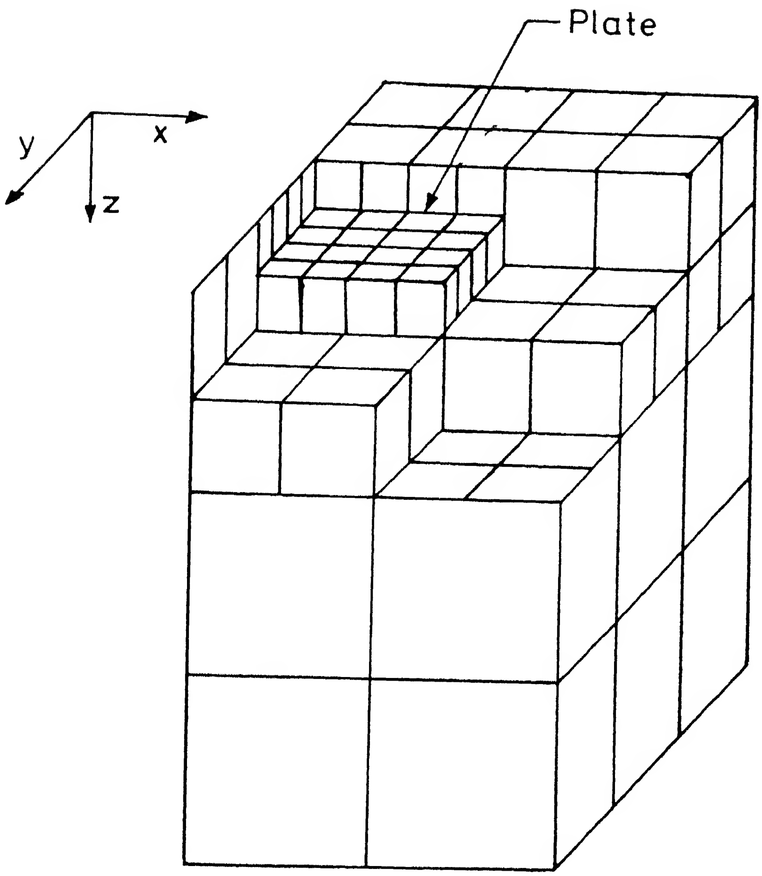


Fig.5 Assembly of eight 8-noded fluid elements.



No. of plate bending elements = 16
No. of 8-noded fluid elements = 72
No. of 13-noded fluid elements = 16
No. of structural d.o.f. =100
No. of fluid d.o.f. =186

Fig.6 Finite element idealisation of plate and fluid.

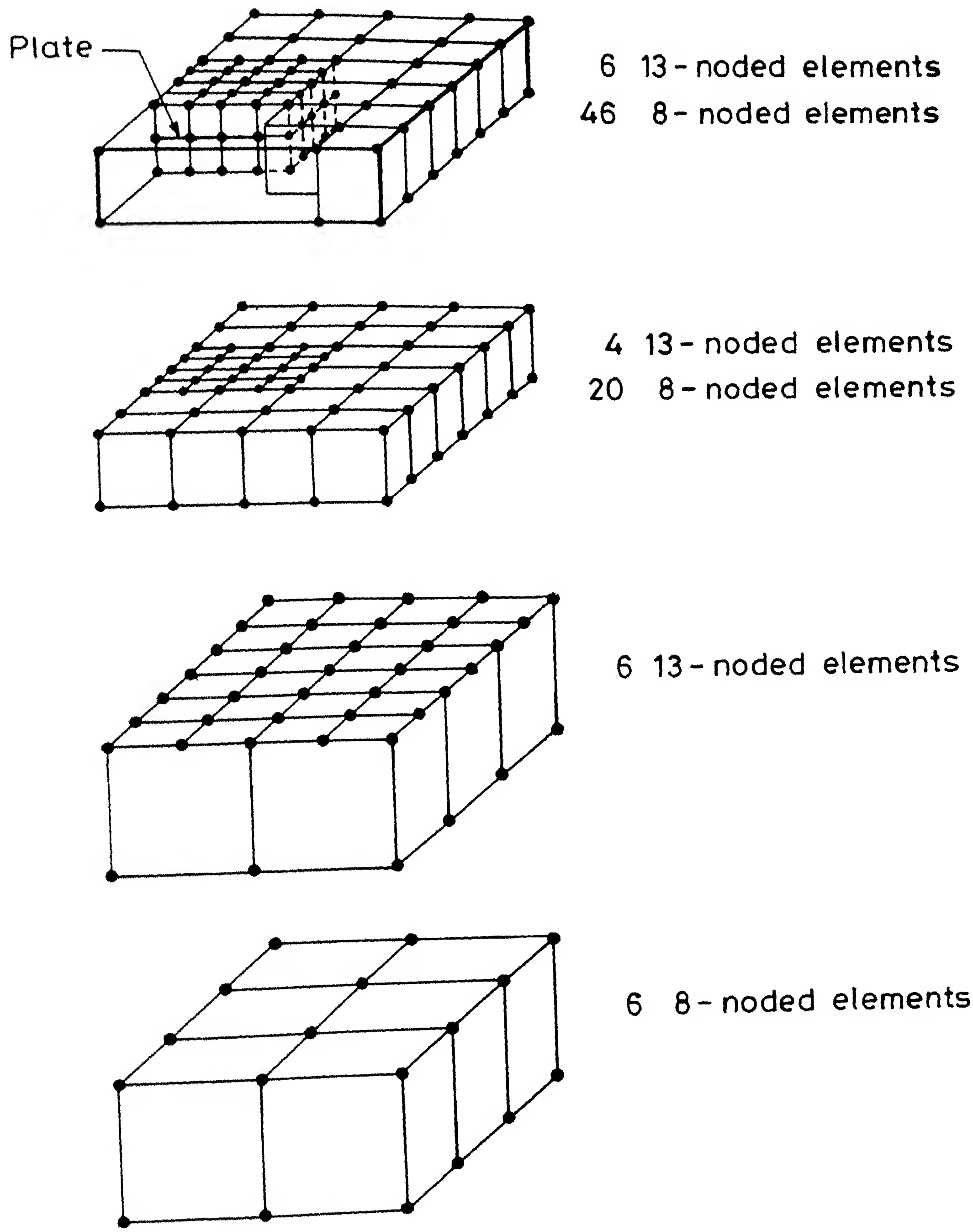


Fig. 7 Layer by layer description of the fluid domain.

CHAPTER- 4

CONCLUSIONS

Based on the results obtained in the previous Chapter the following conclusions are drawn.

(1) The 16 d.o.f. element with implied B.Cs. also is more accurate, computationally more efficient and gives monotonically decreasing convergence.

(2) Condensed 13-noded fluid element gives greater accuracy than the earlier element [16].

(3) The effect of fluid is maximum in the first mode in all laminates studied.

(4) For the first two modes, laminate orientations and the three B.Cs. studied have practically no effect on the percentage decrease in frequencies due to the fluid. The effect on the higher modes due to the fluid show a dependence on the laminate orientations and B.Cs.

(5) Effect of the fluid, as expected, decreases with decrease in fluid density.

(6) The decrease in aspect ratio (b/a) increases the effect of fluid on the frequencies in the first two and fourth modes for the laminate studied. Similar trend is not observed in other two modes.

REFERENCES

1. Muthuveerappan, G., 'Vibrations of elastic structures in fluid environments', Ph.D. Thesis, Dept. of Mech. Engg., I.I.T., Madras, April 1984.

2. Lamb, H., 'On the vibrations of an elastic plate in contact with water', Proc. Roy Soc. (London), Ser. A. V98, p 205-216, 1920.

3. Lewis, F.M., 'The inertia of the water surrounding a vibrating ship', Society of Naval Architects and Marine Engineers, New York, V 37, p 1-20, 1924.

4. Powell, J.H. and Roberts, J.H.T., 'On the frequency of vibration of circular diaphragms', Proc. Phys. Soc. (London) V 35, p 170-72, 1923.

5. Babayev, N.N., 'Investigation of the free vibrations of rectangular plates', Krylov Central Scientific Research Institute, Leningrad, Russia, 1947.

6. Lindholm, Ulric, S., Kana Daniel D., Wen-Hwa Chu and Norman Abramson, H., 'Elastic vibration characteristics of cantilever plates in water', Journal of Ship Research, V 9, n 1, p 11-22, June 1965.

7. Clough, R.W., 'Effect of earthquake on under water structures', Second World Conference on Earthquake Engineering, V 2, p 815, 1966.

8. Malhotra, B.P., 'Determination of Virtual mass of bodies oscillating in water', M.E. Thesis, UOR., 1970.

9. Asghar, J., 'Determination of Virtual mass effect of water on submerged plates', M.E. Thesis, UOR, 1974.

10. Fritz, R.J., 'The effect of liquids on the dynamic motions of immersed solids', Translation of ASME, Journal of Engineering for Industry, V 94, p 167, Feb. 1972.

11. Zienkiewicz, O.C., Iron, B.M., and Nath, B., 'Natural frequencies of complex free or submerged structures by finite element method', Symposium on Vibrations in Civil Engg., London, April'65.

12. Zienkiewicz, O.C. and Newton, R.E., 'Coupled vibrations of a structure submerged in compressible fluid', ISD/ISSC International Symposium on Finite Element Techniques, University of Stuttgart, W.Germany, June 10-12, 1969.

13. Holand, I., 'Finite Elements for the computation of hydrodynamic mass', ISD/ISSC, International Symposium of Finite Element Technique, University of Stuttgart, West Germany, p 509-531, June 10-12, 1969.

14. Nath, B., 'Hydrodynamic pressure on high dams due to vertical earthquake motion', Proceedings of the Institution of Civil Engineering, v 42, p 413. March 1969.

15. Nath, B., 'Coupled hydrodynamics of a gravity dam', IBID., v 48, p 245, Feb. 1971.

16. Chowdhury, P.C., 'Fluid finite elements for added-mass calculations', International ship Building Progress, v 19, n 217, Sept. 1972.

17. Chowdhury, P.C., 'Free Vibrations of fluid-borne Structures'- Investigation on a simple model , North East Coast Institution of Engineers and Ship Builders , p 15-28, 1974.

18. Marcus M.S., 'Finite element applied to the vibration of submerged plates', Journal of Ship Research, v 22, n 2, p 94-95, 1978.

19. Aggarwal, K.R., Sinhasan, R. and Grover, G.K., 'Free vibrations of a rectangular plate submerged in fluid', Institute of Engineers(India), Journal of Mechanical Engineering, v 60, p 184-190, 1980.

20. Muthuveerappan, G. et al, 'Vibration of square cantilever plate immersed in water', J. of Sound and Vibration, v 61, n p 467-470, 1978.

21. Muthuveerappan, G., et al , 'Influence of fluid added mass on the vibration characteristics of plates under various boundary conditions', Journal of Sound and Vibration, v 69, n 4, p 612-615, 1980.

22. Muthuveerappan, G., et al, 'Vibrations of skew plates immersed in water', computers and structures, v 21, n 3, p 479-491, 1985.

23. Muthuveerappan, G., et al, 'Vibration of triangular plates immersed in fluids'; J. Sound and Vibration (paper sent for publication).

24. Jones, R.M., 'Mechanics of Composite Materials', McGraw-Hill Kogakusha Ltd., 1975.

25. Zienkiewicz, O.C., 'The Finite Element Method', Tata McGraw-Hill Publishing Company Ltd., 1979.

26. Gallagher, R.H., 'Finite Element Methods for Engineers', Macmillan, London, 1975.

27. Huebner, K.H., 'The Finite Element Method for Engineers', Wiley- Interscience, New York, 1975.

28. Blevins, R.D., 'Formulas for Natural Frequency and Mode Shapes', Von Nostrand Reinhold Company, 1979.

29. Timoshanko, S.P., and Woinowsky - Krieger, S., 'Theory of plates and shells', McGraw-Hill Kogakusha, 1959.

30. Shames, I.H., Dym, C.L., 'Solid Mechanics : A variational Approach', McGraw-Hill, New York, 1973.